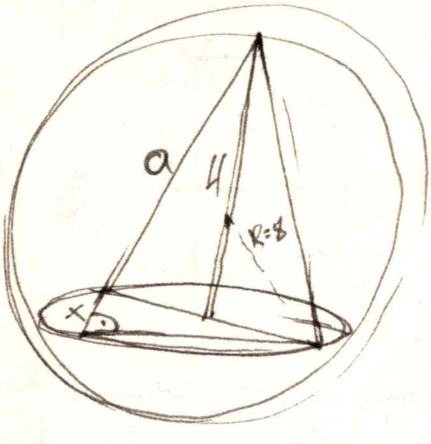
{BRYŁY OBROTOWE}

31.



$H = 12$

$\frac{a\sqrt{3}}{2} = 12$

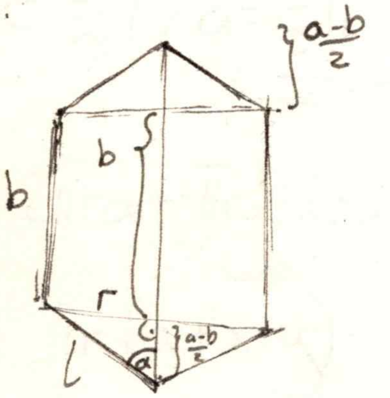
$a = 8\sqrt{3}$

$x\sqrt{2} = 8\sqrt{3}$

$x = 4\sqrt{6}$

$V = \frac{1}{3}x^2 \cdot H = \frac{1}{3} \cdot 16 \cdot 6 \cdot 12 = 192$

32.



$r = \frac{a-b}{2}\operatorname{tg}\alpha$

$l = \frac{a-b}{2\cos\alpha}$

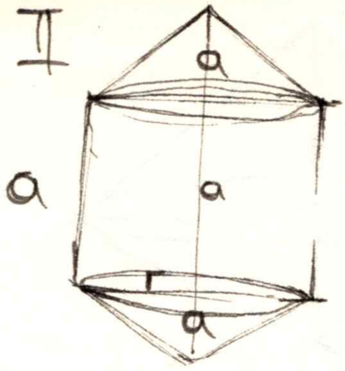
$V = \pi r^2 \cdot b + 2 \cdot \frac{1}{3}\pi r^2 \cdot \frac{a-b}{2} =$

$= \pi r^2\left(b + \frac{2}{3} \cdot \frac{a-b}{2}\right) = \pi \frac{(a-b)^2}{4} \cdot \frac{2b+3a}{3}\operatorname{tg}\alpha$

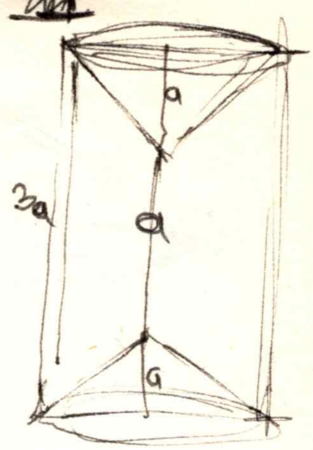
$P_c = 2\pi r l + 2\pi r b = 2\pi r(l+b) = \pi(a-b)\operatorname{tg}\alpha \cdot \left(\frac{a-b}{2\cos\alpha} + b\right)$

33.

II



I



$V_{II} = \pi r^2 \cdot a + 2 \cdot \frac{1}{3}\pi r^2 \cdot a$

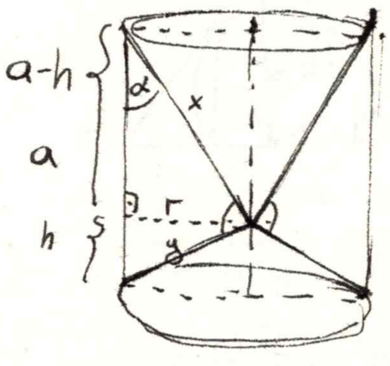
$V_{II} = \frac{5}{3}\pi r^2 \cdot a$

$V_I = \pi r^2 \cdot 3a - 2 \cdot \frac{1}{3}\pi r^2 a$

$V_I = \frac{7}{3}\pi r^2 \cdot a$

$\frac{V_I}{V_{II}} = \frac{7}{5}$

34.



$\cos\alpha = \frac{x}{a}$

$x = a\cos\alpha$

$\sin\alpha = \frac{r}{x}$

$r = x\sin\alpha = a\sin\alpha\cos\alpha =$

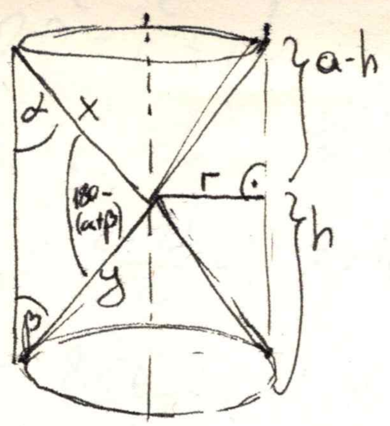
$r = \frac{1}{2}a\sin 2\alpha$

$V = \pi r^2 \cdot a - \frac{1}{3}\pi r^2 \cdot h - \frac{1}{3}\pi r^2 (a-h) =$

$= \frac{2}{3}\pi r^2 \cdot a = \frac{2}{3}\pi \cdot \left(\frac{1}{2}a\sin 2\alpha\right)^2 \cdot a =$

$= \frac{a^3 \sin 2\alpha}{6}$

35.



$\frac{a}{\sin(180-(\alpha+\beta))} = \frac{x}{\sin\beta}$ $\quad$ $\frac{a}{\sin(180-(\alpha+\beta))} = \frac{y}{\sin\alpha}$

$x = \frac{a\sin\beta}{\sin(\alpha+\beta)}$ $\quad$ $y = \frac{a\sin\alpha}{\sin(\alpha+\beta)}$

$P_\Delta = \cancel{\frac{1}{2}} a\cdot x\cdot\sin\alpha = \cancel{\frac{1}{2}} r\cdot a$

$r = x\sin\alpha = \frac{a\sin\beta\sin\alpha}{\sin(\alpha+\beta)}$

$V = \pi r^2\cdot a - \frac{1}{3}\pi r^2\cdot h - \frac{1}{3}\pi r^2(a-h) =$

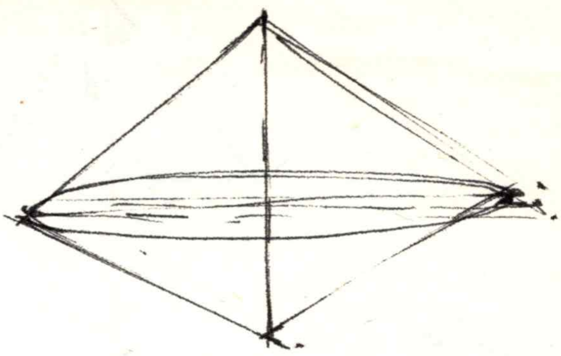
$= \frac{2}{3}\pi r^2 a = \frac{2}{3}\pi\cdot\frac{a^3\sin^2\beta\sin^2\beta}{\sin^2(\alpha+\beta)}$

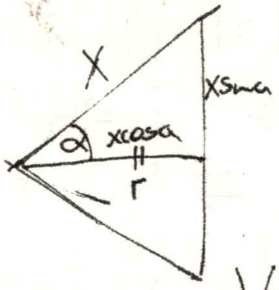
$P_c = 2\pi r a + \pi r y + \pi r x =$

$= \pi r(2a + x + y) =$

$= \pi\cdot\frac{a\sin\beta\sin\alpha}{\sin(\alpha+\beta)}\left(2a + \frac{a\sin\beta}{\sin(\alpha+\beta)} + \frac{a\sin\alpha}{\sin(\alpha+\beta)}\right)$

36





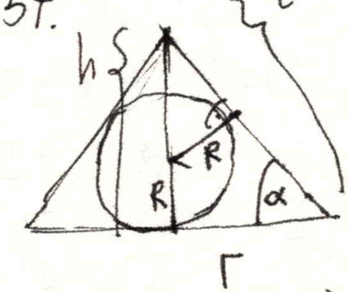
$2x + 2x\sin\alpha = P$

$x = \frac{P}{2+2\sin\alpha}$

$V = 2\cdot\frac{1}{3}\pi r^2\cdot x\sin\alpha =$

$= \frac{2}{3}\pi x^3\cos^2\alpha\sin\alpha$

37.



$\cos\alpha = ?\ \frac{r}{l}$

$(\pi r^2, 4\pi R^2, \pi r l)$ – c. arytm.

$8\pi R^2 = \pi r^2 + \pi r l$

$\begin{cases} 8R^2 = r^2 + rl \\ h^2 + r^2 = l^2 \\ \cancel{hr = \frac{2r\cdot l^2}{4R}} \end{cases}$ → Pitagoras
– Pole Δ

→ Kolejna strona

$\cancel{h = \frac{l^2}{2R}}$

$\cancel{\begin{cases} \frac{l^4}{4R^2} + r^2 = l^2 \\ 8R^2 = r^2 + rl \end{cases}}$

$\cancel{2l^4 = l^2 - r^2}$

$\cancel{2l^4 = (r^2 + rl)(l^2 - r^2) : l^4}$

$\cancel{2 = \left(\frac{r^2}{l^2} + \frac{r}{l}\right)\left(1 - \frac{r^2}{l^2}\right)}$

$\cancel{t = \frac{r}{l}}$

$\cancel{2 = (t^2 + t)(1 - t^2)}$

37.

$$\mathrm{I}\begin{cases} 8R^2 = r^2 + rl \\ h^2 + r^2 = l^2 \Rightarrow h^2 = l^2 - r^2 \\ hr = \frac{2r+2l}{2} \circ R \Rightarrow R = \frac{hr}{r+l} \end{cases}$$

z I $\quad \frac{8h^2r^2}{(r+l)^2} = r(r+l)$

$$\frac{8(l^2-r^2)r^2}{(r+l)^2} = r(r+l)$$

$$\frac{8(l-r)r}{(r+l)} = (r+l)$$

$$8r(l-r) = r^2 + 2rl + l^2$$

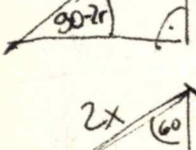
$$9r^2 - 6rl + l^2 = 0$$

$$(3r - l)^2 = 0$$

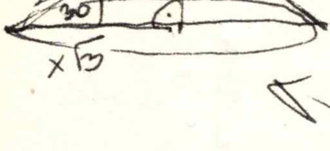
$$l = 3r$$

$$\cos\alpha = \frac{r}{l} = \frac{r}{3r} = \frac{1}{3}$$

38.



$$90 - r + 90 - 2r = 90$$
$$r = 30$$

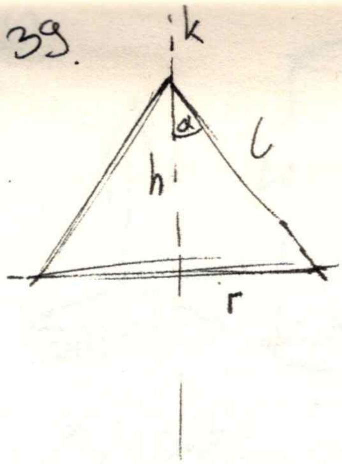


$$3x + x\sqrt{3} = 3(\sqrt{6} + \sqrt{2})$$
$$x(3+\sqrt{3}) = 3(\sqrt{6}+\sqrt{2})$$
$$x = \frac{3(\sqrt{6}+\sqrt{2})}{3+\sqrt{3}}$$

Odbi już odpowiedzio

$$V = \frac{1}{3}\pi(x\sqrt{3})^2 \cdot x = \pi x^3 = \pi\left(\frac{3(\sqrt{6}+\sqrt{2})}{3+\sqrt{3}}\right)^3$$

39.



$\pi r^2 + \pi rl = \pi\sqrt{3} \cdot hr \Leftarrow$ z treści

$$\begin{cases} r^2 + rl = \sqrt{3}hr \Rightarrow r + l = \sqrt{3}h \\ h^2 + r^2 = l^2 \Leftarrow \text{Pitagoras} \end{cases}$$

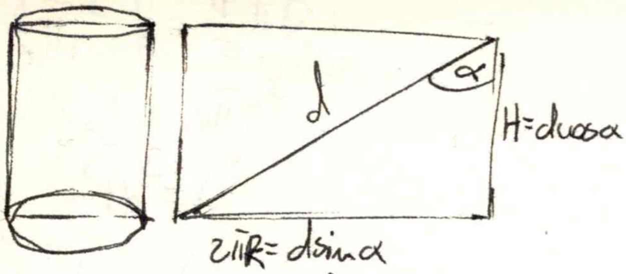
$$\begin{cases} r^2 + 2rl + l^2 = 3h^2 \\ h^2 = l^2 - r^2 \end{cases}$$

$$r^2 + 2rl + l^2 = 3l^2 - 3r^2$$
$$4r^2 + 2rl - 2l^2 = 0$$
$$2r^2 + rl - l^2 = 0$$
$$\Delta = l^2 + 8l^2$$
$$r_1 = \frac{-l+3l}{4} = \frac{1}{2}l$$
$$r_2 = \frac{-l-3l}{4} = -l \ ↯$$

$$r = \frac{1}{2}l$$

$$\sin\alpha = \frac{r}{l} = \frac{1}{2} \Rightarrow \alpha = 30^\circ$$

40



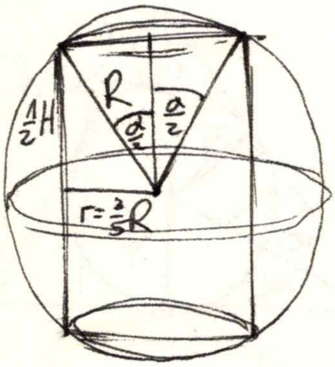
$2\pi R = d\sin\alpha$

$R = \frac{d\sin\alpha}{2\pi}$

$V = \pi R^2 \cdot H = \pi \cdot \frac{d^2 \sin^2\alpha}{4\pi^2} \cdot d\cos\alpha =$

$= \frac{d^3 \sin^2\alpha \cos\alpha}{4\pi}$

41.



$\left(\frac{1}{2}H\right)^2 + \left(\frac{3}{5}R\right)^2 = R^2$

$\left(\frac{1}{2}H\right)^2 = \frac{16}{25}R^2$

$H = \frac{8}{5}R$

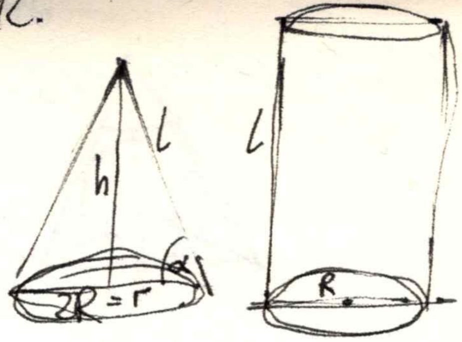
$\frac{V_w}{V_k} = \frac{\cancel{\pi}\left(\frac{3}{5}R\right)^2 \cdot \frac{8}{5}R}{\frac{4}{3}\cancel{\pi}R^3} = \frac{54}{125}$

$\sin\frac{\alpha}{2} = \frac{3}{5}$

$\cos\frac{\alpha}{2} = \frac{4}{5}$

$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$

42.



1) $\cancel{\pi} r \cancel{l} = 2\cancel{\pi} R \cancel{l}$

$r = 2R$

2) $\frac{1}{3}\pi r^2 \cdot h = \pi R^2 \cdot l$

$\frac{1}{3} \cdot 4\cancel{R^2} \cdot h = \cancel{R^2} \cdot l$

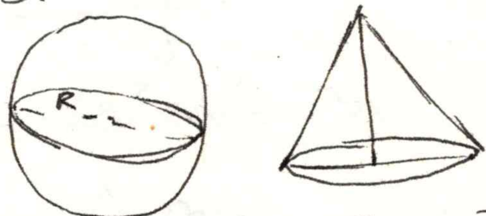
$h = \frac{3}{4}l$

$\sin\alpha = \frac{h}{l} = \frac{3}{4}$

$\cos^2\alpha = 1 - \sin^2\alpha = \frac{7}{16}$

$\cos\alpha = \frac{\sqrt{7}}{4}$

43.



$\frac{1}{3}\pi r^2 \cdot h = \frac{4}{3}\pi R^3$ $\qquad$ $\pi r l = 3\pi r^2$

$l = 3r$

$\begin{cases} r^2 h = 4R^3 \\ l = 3r \\ h^2 + r^2 = l^2 \end{cases}$

$h^2 = 8r^2$

$\frac{h^3}{8} = 4R^3$

$h = 2\sqrt[3]{4}R$

44

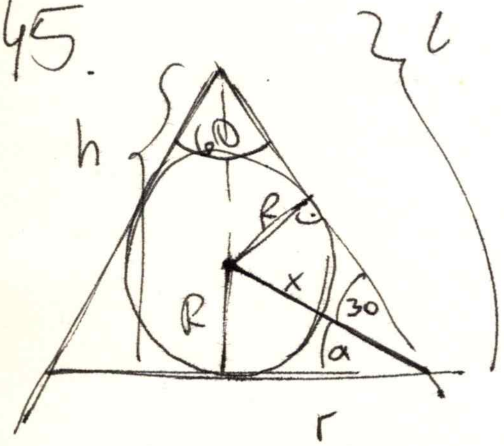
$$\begin{cases} \frac{1}{3}\pi r^2 \cdot h = \frac{4}{3}\pi R^3 \\ \pi r l = 3\pi r^2 \\ r^2 + h^2 = l^2 \end{cases}$$

$$\begin{cases} r^2 h = 4R^3 \\ l = 3r \\ h^2 = 8r^2 \Rightarrow r = \frac{h}{2\sqrt{2}} \end{cases}$$

$h = \cancel{8R}\ 2\sqrt[3]{4}\,R$

$r = \frac{\cancel{2}\sqrt[3]{4}R}{\cancel{2}\sqrt{2}} = \frac{\sqrt[3]{4}R}{\sqrt{2}} = \sqrt[6]{2}\,R$

45.



$\cancel{\frac{\pi r^2}{4\pi R^2} = \frac{3}{4}}$

$\cancel{r^2 = \frac{3}{4}R^2}$

$\frac{\pi r^2}{4\pi R^2} = \frac{3}{4}$

$r^2 = 3R^2$

$r = R\sqrt{3}$

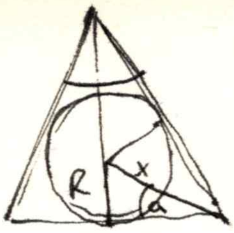
$x^2 = r^2 + R^2 \Rightarrow x = 2R$

$\sin\alpha = \frac{R}{2R} = \frac{1}{2} \Rightarrow \alpha = 30$

$h = 3R$

a) 60° b) $\frac{V_k}{V_s} = \frac{\frac{4}{3}\pi R^3}{\frac{1}{3}\pi \cdot 3R^2 \cdot 3R} = \frac{4}{9}$ c) $\frac{P_{cs}}{P_k} = \frac{\pi \cdot 3R^2 + \pi R\sqrt{3} \cdot 2R\sqrt{3}}{4\pi R^2} = \frac{9}{4}$

46.



$\frac{4\pi R^2}{\pi r^2} = \frac{16}{9}$

$R = \frac{2}{3}r$

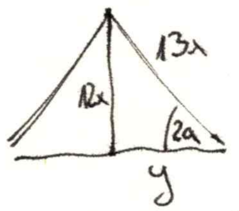
$x^2 = R^2 + r^2 = \frac{13}{9}r^2$

$x = r\frac{\sqrt{13}}{3}$

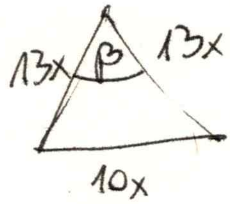
$\cos\alpha = \frac{3}{\sqrt{13}}$

$\sin\alpha = \frac{2}{\sqrt{13}}$

$\sin 2\alpha = \frac{12}{13}$



$y^2 = 169x^2 - 144x^2 \Rightarrow y = 5x$



$(10x)^2 = (13x)^2 + (13x)^2 - 2 \cdot 13x \cdot 13x \cdot \cos\beta$

$100x^2 = 338x^2 - 338x^2\cos\beta$

$\cos\beta = \frac{228}{338} = \frac{114}{169}$

$\sin\beta = \frac{\sqrt{15565}}{169}$